Chapter Section 7.1 Integration by Parts

Traditional $\int u\,dv = uv - \int v\,du$

Ex: Using this format $\int x(x-2)^5\,dx$

Step 1 traditional u-substitution

$u = x-2$
$du = dx$

Problem is that $\int (x-2)^5$ $\underline{\underline{x}}\,dx$

extra (x), so does not work traditionally.

Step 2 $u = x$ $dv = \underbrace{(x-2)^5}\,dx$
$du = dx$

we can integrate

$v = \int (x-2)^5\,dx$
$= \frac{(x-2)^6}{6}$ this removes b/c integrated

Step 3 $\int u\,dv = uv - \int v\,du = \frac{x(x-2)^6}{6} - \int \frac{(x-2)^6}{6}\,dx$

Should be integratable now if done correctly.

Step 4 $= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{6\cdot 7} + C$

Step 5 $\boxed{= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + C}$

* Same as previous answer of $\frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C$

* To double check answer choose some test points and see if the two answers are the same
* use $x = 1$ or $x = \emptyset$

, so $\dfrac{1(-1)^6}{6} - \dfrac{(1-2)^7}{42} = \dfrac{1}{6} \; 1 \text{-} \dfrac{1}{42} = \boxed{0.1905}$ ✓

, and $\dfrac{(1-2)^7}{7} + \dfrac{(1-2)^6}{3} = \dfrac{-1}{7} + \dfrac{1}{3} = \boxed{0.1905}$ ✓

Shortcut Method:

$$\int x(x-2)^5\,dx$$

Straight u-Substition
$u = x-2$
$du = dx$
$\Rightarrow$ extra(x)

	D		I
+	x		$(x-2)^5\,dx$
− derivative of x	1	straight derivative	$\dfrac{(x-2)^6}{6}$
+ derivative of 1	$\emptyset$	derivative again!!	$\dfrac{(x-2)^7}{6*7}$

alternate signs from top to bottom

Cross down; @ last one it goes across

$$= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + \int \frac{0 \cdot (x-2)^7}{42}\,dx$$

$\emptyset$

$= +C$

$$\boxed{= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + C}$$

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Ex: $\int \frac{2x+1}{e^{3x}}\,dx$ ~~u =~~

If denominator can move to numerator then it can/cannot be your (u).

or $\int (2x+1)\, e^{-3x}\,dx$

If denominator is one piece - move to numerator, if has more than one part i.e. $(1+e^{3x})^4$ then use as (u) value.

, so $u=-3x$
$du=-3\,dx$

$= \int e^{-3x} \underbrace{(2x+1)\,dx}$

b/c $du=-3dx$ and does not have a $(2x+1)$ have to use integration by parts.

, so

	D		I
+	$(2x+1)$	*b/c this is where extra x is	e^{-3x} $u=-3x$ $du=-3dx$
−	derivative of $(2x+1)$: 2		$-\frac{1}{3}e^{-3x}$
+	derivative of (2): $\int 0$		$\frac{1}{9}e^{-3x}$

1st pc, 2nd pc, 3rd pc

$$\boxed{= -\frac{(2x+1)}{3}e^{-3x} - \frac{2}{9}e^{-3x} + C}$$

Ex: $\int x^2 \ln x \, dx$

Step 1 1st choice – Does not work
$\cancel{u = x^2}$
$\cancel{du = 2x\,dx}$

Step 2 2nd choice
$\cancel{u = \ln x}$
$\cancel{du = \frac{1}{x}\,dx}$
Does not work

$= \int \ln x \; x^2\,dx$ or $\int x^2 \; \ln x\,dx$

Do by parts b/c neither (u)-sub. works

Step 3

D	I
x^2	$\ln x$
$2x$	? Don't know $\ln x$ integral
2	
ϕ	

So swap D ; I

Step 4

D	I
+ $\ln x$	x^2
− $\int \frac{1}{x} = x^{-1}$	$\frac{x^3}{3}$
$-x^{-2}$	

Step 5

* always begin Step 5 – with + − + in this format ☺

Step6 $\ln x \cdot \frac{x^3}{3} - \int \left(\frac{1}{x}\right)\left(\frac{x^3}{3}\right) dx$

ALWAYS Last part of D and I sorting

Step7 $\frac{x^3}{3} \cdot \ln x - \frac{1}{3}\int x^2 dx$

Step8 Integrate $= \frac{x^3}{3}\ln x - \frac{1}{3}\cdot\frac{x^3}{3} + C$

Step9 $\boxed{\frac{x^3}{3}\ln x - \frac{1}{9}x^3 + C}$ Final answer!! :)

Ex: $\int \frac{(\ln x)^7}{x} dx$

$u = \ln x$
$du = \frac{1}{x} dx$

$= \int (\ln x)^7 \cdot \frac{1}{x} dx$

$\boxed{= \frac{(\ln x)^8}{8} + C}$

Similar to one on test **

Remember!! :)