

Chapter Section 7.1 | Integration by Parts

Traditional

$$u dv = uv - \int v du$$

Ex: Using this format

$$\int x(x-2)^5 dx$$

Step 1 traditional u-substitution $u = x-2$
 $du = dx$

Problem is that $\int (x-2)^5 \underline{x} dx$

extra(x), so does not work traditionally.

Step 2 $u = x$
 $du = dx$

$$dv = (x-2)^5 dx$$

We can integrate

$$v = \int (x-2)^5 dx$$

$$= \frac{(x-2)^6}{6}$$

this removes b/c integrated

Step 3 $\int u dv = uv - \int v du = \frac{x(x-2)^6}{6} - \int \frac{(x-2)^6}{6} dx$

Should be integratable now if done correctly.

Step 4 $= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{6 \cdot 7} + C$

Step 5 $= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + C$

* Same as previous answer of $\frac{(x-2)^7}{7} + \frac{(x-2)^6}{3} + C$

* To double check answer choose some test points and see if the two answers are the same

* use $x=1$ or $x=0$

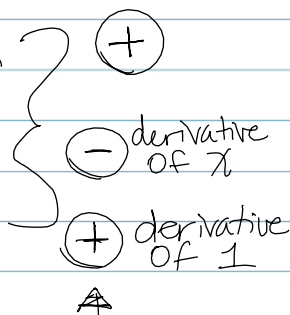
so $\frac{1(-1)^6}{6} - \frac{(-1-2)^7}{42} = \frac{1}{6} - \frac{1}{42} = \boxed{0.1905}$

and $\frac{(1-2)^7}{7} + \frac{(1-2)^6}{3} = -\frac{1}{7} + \frac{1}{3} = \boxed{0.1905}$

Shortcut Method:

$\int x(x-2)^5 dx$

Straight u-substitution
 $u = x-2$
 $du = dx$
 \Rightarrow extra(x)



alternate signs from top to bottom

D

x

1

\emptyset

I

$(x-2)^5 dx$

straight derivative $\frac{(x-2)^6}{6}$

derivative again!! $\frac{(x-2)^7}{6 \cdot 7}$

Cross down @ last one it goes across

$= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + \int \frac{0 \cdot (x-2)^7}{42} dx$

$C = + C$

$= \frac{x(x-2)^6}{6} - \frac{(x-2)^7}{42} + C$

Ex:

$$\int \frac{2x+1}{e^{3x}} dx$$

~~u =~~

If denominator can move to numerator then ~~it~~ ^{it can/cannot} be your (u).

or

$$\int (2x+1) e^{-3x} dx$$

If denominator is one piece - move to numerator, if have more than one part i.e. $(1+e^{3x})^4$ then use as (u) value.

so $u = -3x$
 $du = -3dx$

$$= \int e^{-3x} (2x+1) dx$$

b/c $du = -3dx$ and does not have a $(2x+1)$ have to use integration by parts.

<p>(+) SO</p> <p>(-) derivative of $(2x+1)$</p> <p>(+) derivative of $\int \phi$</p>	<p>D</p> <p>$(2x+1)$</p> <p>2</p> <p>$\int \phi$</p>	<p>*b/c this is where the extra x is</p>	<p>I</p> <p>e^{-3x}</p> <p>$u = -3x$ $du = -3dx$</p> <p>$-\frac{1}{3}e^{-3x}$</p> <p>$\frac{1}{9}e^{-3x}$</p>
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1 st PC	2 nd PC	3 rd PC
$-\frac{(2x+1)}{3} e^{-3x}$	$-\frac{2}{9} e^{-3x}$	$+ C$

Ex:

$$\int x^2 \ln x \, dx$$

Step 1

1st choice - Does not work

~~$$u = x^2$$

$$du = 2x \, dx$$~~

Step 2

2nd choice - Does not work

~~$$u = \ln x$$

$$du = \frac{1}{x} \, dx$$~~

$$= \int \ln x \, \underline{\underline{x^2 \, dx}} \quad \text{or} \quad \int x^2 \, \underline{\underline{\ln x \, dx}}$$

Do by parts b/c neither (u)-sub. works

Step 3

$$\frac{1}{x^2}$$

$$2x$$

$$2$$

$$\emptyset$$

$$\frac{1}{\ln x}$$

? Don't know lnx integral

So swap D & I

Step 4

$$D$$

$$+ \ln x$$

$$I$$

$$x^2$$

Step 5

$$- \int \frac{1}{x} = x^{-1}$$

$$-x^{-2}$$

$$\frac{x^3}{3}$$

* always begin Step 5 - with (+) (-) (+) in this format ☺

Step 6

$$\ln x \cdot \frac{x^3}{3} - \int \left(\frac{1}{x} \right) \left(\frac{x^3}{3} \right) dx$$

ALWAYS Last part of D and I sorting

Step 7

$$\frac{x^3}{3} \cdot \ln x - \frac{1}{3} \int x^2 dx$$

Step 8

Integrate $= \frac{x^3}{3} \ln x - \frac{1}{3} \cdot \frac{x^3}{3} + C$

Step 9

$$\frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C$$

Final answer !!

Ex:

$$\int \frac{(\ln x)^7}{x} dx$$

$$u = \ln x \\ du = \frac{1}{x} dx$$

$$= \int (\ln x)^7 \cdot \frac{1}{x} dx$$

$$= \frac{(\ln x)^8}{8} + C$$

Similar to one on test

Remember !!